Searching for Chaos in Fibrillation[a]

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INTRODUCTION

Ventricular fibrillation (VF) is a state of disorganized, ineffective contraction of the heart's two main pumping chambers. Ventricular fibrillation has traditionally been described as "turbulent" and chaotic,[1] and in recent years there has been speculation that fibrillation may be an instance of deterministic chaos in the context of nonlinear dynamical systems theory.

The idea that fibrillation is chaos has received indirect support both from mathematical models and experimental observations. A simple deterministic[2–4] model of cardiac electrical activity displays fibrillation-like activity. This suggests that the seeming randomness of fibrillation may arise from a deterministic dynamical system.

Experiments oriented toward investigating the transition from a normal heart rhythm to fibrillation have shown that there is a correlation between decreased electrical stability of the heart and a "period doubling" in cardiac rhythm.[5] The experiments begin with the heart in an approximately periodic rhythm. The changes in this rhythm are observed as a parameter of the system—such as temperature—is changed. At several points in the experiment, the stability of the pattern of electrical activity in the heart is probed by applying a series of electrical perturbations to the heart. The amount of electrical current needed to induce VF is taken as a measure of electrical stability: the greater the required current, the greater the stability.

It is tempting to see such experiments as analogous to numerical "experiments" in which a parameter of a dynamical system is gradually changed, and the resulting bifurcation behavior is observed. One widely observed route to chaos in such numerical experiments is a cascade of period doublings that turn a periodic system into an aperiodic and chaotic one.

Although the observed doubling in the period of the heart's rhythm may be such a period doubling on the way to chaos, the analogy between the cardiac electrical instability experiments and the numerical experiments is not perfect. In particular, the type of stability tested in the cardiac experiments has to do with the proximity of different basins of attraction, and not the pitchfork bifurcations associated with the period-doubling route to chaos.[6,7]

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MAKING A DYNAMICAL SYSTEMS REPRESENTATION OF FIBRILLATION

In order to test directly the idea that fibrillation is chaos, we sought to construct a dynamical system capable of representing the surface electrocardiographic (ECG) data collected during fibrillation. This dynamical system can be tested for the attributes of chaos, such as the existence of an attractor and sensitive dependence on initial conditions.

The general approach in constructing such a representation is to embed the ECG signal in a space using the method of lags.[8] In the method of lags, a scalar signal $s(t)$ is

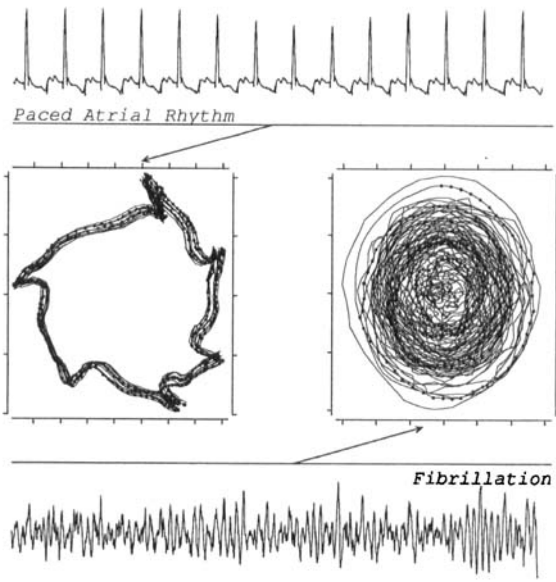

FIGURE 1. State-space reconstructions of 3-lead ECGs from an atrially paced rhythm and from fibrillation. A projection of the trajectory onto two dimensions is shown. The trajectory plotted covers 5 s, while the *black dots* indicate the trajectory during a short subsection of that period.

used to generate an m-dimensional vector $\mathbf{s}(t) \equiv (s(t), s(t-\tau), s(t-2\tau), \ldots, s(t-(m-1)\tau))$, where τ is a lag time chosen by the researcher. (The method can be easily applied as well to vector signals $(s_1(t), s_2(t), \ldots, s_l(t))$.) In the m-dimensional space, this vector parameterized by time constitutes a trajectory.

The m-dimensional space can be considered a putative state-space, and the trajectory tested to see if it is consistent with a deterministic flow in that state-space.

Any trajectory is consistent with a deterministic flow if the trajectory does not cross itself. For $m \geq 3$, however, one cannot expect any finite length of trajectory actually to cross itself. In general, the larger m is, the larger the length of signal needed to produce a crossing. If the system is not deterministic, there will be near-crossings where two branches of the trajectory are approximately orthogonal. The length scale that determines what constitutes a "near"-crossing is that imposed by the noise in the experimental system.

FIGURE 1 shows a two-dimensional projection of a 21-dimensional embedding of three-lead ECGs from a paced atrial rhythm and fibrillation. The trajectory of the paced atrial rhythm is ring-shaped, corresponding to the regular, periodic ECG. The ring is thick—its thickness is produced by noisy dynamics of the system at short length scales. Not all the dynamics at the short length scales, however, are due to noise. In this particular case, recorded a few seconds before the onset of fibrillation, the ring is split down the middle, and the trajectory alternately courses through one side and then the other of the split. This alternation is the state-space manifestation of a doubled period in the ECG.

In contrast, fibrillation appears to be a tangled knot. The trajectory is crossing itself in many places in this two-dimensional projection. It is not clear that there is a length scale that separates the noisy dynamics from the deterministic ones. This two-dimensional plot may not, however, accurately represent the situation in the full 21-dimensional space. In that larger space, the dynamics may be untangled.

USING THE DIMENSION TO MONITOR THE UNTANGLING OF DYNAMICS

Since the trajectory is a geometrical object, its dimension can be calculated. The dimension of the trajectory allows us to investigate the existence of near-crossings without resorting to plots of the sort in FIGURE 1. The method is as follows. Embed the signal in a putative state-space. Calculate the dimension of the trajectory. Embed the signal in a somewhat larger dimensional putative state-space. Again, calculate the dimension of the trajectory in the larger dimensional state-space. Continue this process until the calculated dimension is independent of the embedding dimension.

So long as the trajectory looks like a tangled knot, the trajectory's dimension will increase with the dimension of the embedding space. When there are near-crossings that are approximately orthogonal, the larger dimensional embedding will tend to separate the two branches of the trajectory involved in the crossing. When this "untangling" is complete, the trajectory's dimension will stop increasing with embedding dimension.

When the dimension stops increasing with embedding dimension, we can be satisfied that there are no approximately orthogonal crossings left in the trajectory, and that we have a deterministic state-space representation of the signal from which we created the trajectory.

Unfortunately, for any finite-length signal, there is some embedding that will untangle the trajectory. In order to be assured that the success of the embedding is due to the deterministic dynamics of the system that generated the trajectory, and not due to the finite length of the signal, we need to compare the dimension of the signal of interest to the dimension of a random, but similar signal of the same length.

It is unclear what is the best definition of "similar" for this purpose. In our work, we have taken two signals to be similar when they have the same power spectrum. It is possible to generate a random signal with an identical power spectrum to a signal of interest by randomizing the phases in the Fourier representation of the signal. That is, our random signal generator consists of taking the FFT of the signal of interest, randomizing the phases uniformly over the interval $[-\pi, \pi]$, and taking the inverse FFT.

The most widely used technique for calculating dimensions is the Grassberger–Procaccia method,[9] which considers how the number of pairs of points closer than a certain distance changes with that distance. That is, let $C(l)$ be the number of pairs of points in the trajectory closer together than l. The Grassberger–Procaccia dimension is

$$\nu_{gp}(l) = \frac{d \log C(l)}{d \log l}.$$

Another possible method for calculating dimension is the "box-counting" method, which covers the trajectory with hypercubes. Let $N(l)$ be the smallest number of m-dimensional hypercubes needed to cover completely the trajectory embedded in a m-dimensional space. The dimension is

$$\nu_{bc}(l) = \frac{d \log N(l)}{d \log l}.$$

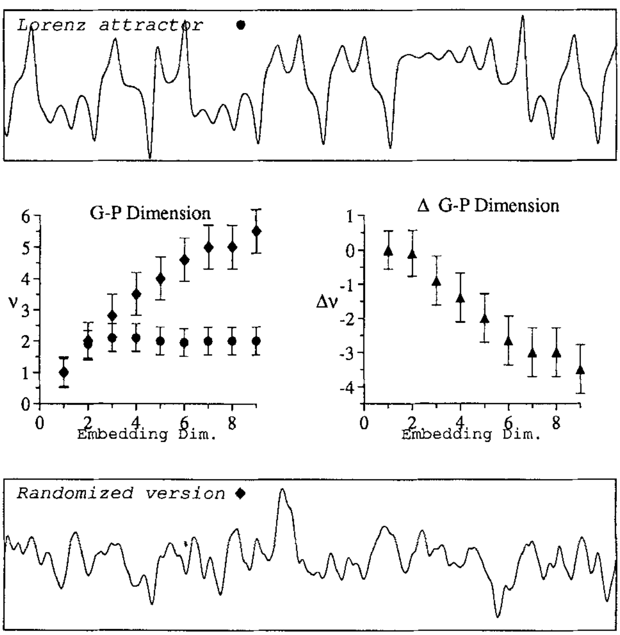

FIGURE 2. The calculated dimension, ν of a trajectory based on a signal derived from the Lorenz equations, and a randomized version of the signal. The difference between ν for the two signals is also shown. The error bars reflect the uncertainty in the estimate of the dimension, for which 8000 points were used (1000 are shown in the figure). Although the power spectra of the Lorenz signal and the randomized signal are identical, the signals have very different appearances.

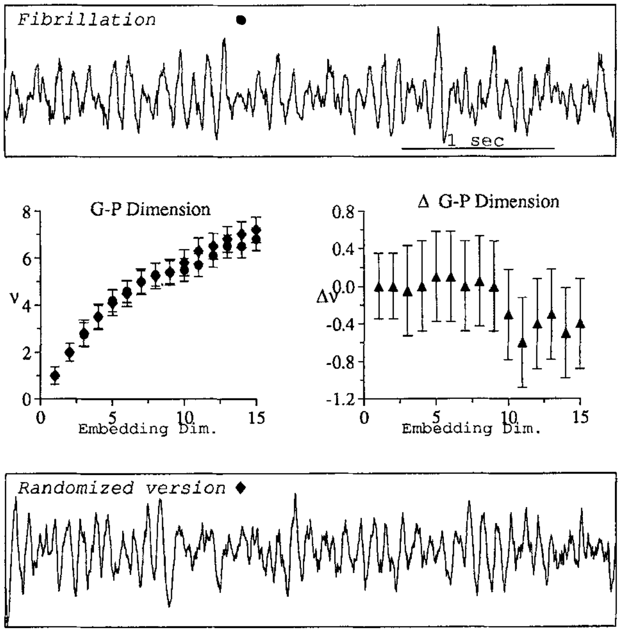

FIGURE 3. The calculated dimension of the trajectory of a fibrillatory ECG and a random signal with the same power spectrum as the ECG. The original signal and the randomized version are qualitatively similar. The dimension calculations bear this out: there is little difference between the dimension of the random signal and that of the fibrillation signal.

The two methods generally give similar results; ν_{gp} has been preferred because of the computational efficiency of its calculation. Recently, however, the box-counting method has been given a computer-efficient formulation by Liebovitch and Toth.[10] Although we used ν_{gp} in our work, the box-counting method may prove superior, since ν_{gp} is strongly affected by the finite extent of the object whose dimension is being calculated.

Both ν_{gp} and ν_{bc} give dimension as a function of length scale. This is useful, since noise is a part of any experimental system, and it is often possible to find a length scale that separates noise from the dynamics of interest. The question, however, is still open of how to choose a length scale that is representative of a dynamical system. A common approach is to look for length scales at which the dimension is comparatively independent of length scale, that is, plateaus.

FIGURE 2 shows a signal generated from the Lorenz equations, as well as a randomized version of the same signal. The calculated dimension of the trajectory for each of the two signals for different embedding dimensions is shown. For embedding dimensions greater than 2, the dimension of the Lorenz attractor is approximately 2. The dimension of the trajectory derived from the randomized signal increases with the embedding dimension. From this we can see that the finite length of the signal used in the calculation does not play a role in the leveling off of the graph of trajectory dimension vs. embedding dimension for the Lorenz equations.

The situation is different in FIGURE 3, which shows one lead of an ECG from fibrillation, and the corresponding randomized signal. The dimension of the fibrillation trajectory is not very different from the dimension of the randomized signal. While it is clear that the dimension of both signals is asymptotically approaching approximately 8, this cannot really be taken to mean that the dynamics of fibrillation have an eight-dimensional attractor. The saturation of the dimension seems to be due to the finite length of the signal used and the limited spectral content of fibrillation.

IS FIBRILLATION CHAOS?

To address the question of whether fibrillation is chaos, we applied the technique previously outlined to analyze ECGs from ventricular fibrillation in dogs. This section summarizes the results presented in reference 11. The data were collected in a series of experiments performed by Smith *et al.* and reported in reference 5. Four episodes from four different dogs were studied. The episodes of VF were uninterrupted by attempts to modify or terminate the fibrillation, and the dogs were maintained on mechanical respiration during the fibrillation episode, allowing respiratory artifacts to be effectively eliminated from the signal by high-pass filtering.

Electrocardiograms were collected from three approximately orthogonal leads, recorded first in analog form on FM tape, and then digitized at 1000 Hz after appropriate antialiasing filtering. The digitized data were digitally filtered to remove respiration artifact with a 1027-point, zero phase-shift filter with cutoff at 2 Hz. In all cases, the spectrum of the unfiltered signal had a large peak at respiration frequency (usually 0.25 Hz, corresponding to a mechanical respiratory rate of 15 breaths/minute), and very little spectral content between 1 and 4 Hz.

For each episode of VF, several signal segments were generated from an 8-s-long segment starting 1 to 2 s after the change from a tachycardia-like rhythm to fibrillation. The segments ranged from 1 to 8 s in length, at sampling rates of 62.5 Hz, 125 Hz, 250 Hz, and 1000 Hz.

The segments were embedded in a 21-dimensional space using the method of lags, for delay time τ ranging from 30 to 100 ms. The method of principal components was used to select smaller dimensional spaces from the 21-dimensional space. To make an n-dimensional putative state-space, the n largest principal components of the cloud of points in the 21-dimensional space were selected.[12] The dimension of the trajectory was calculated using the Grassberger–Procaccia method.[9,13] The same calculations were performed on randomized versions of each of the segments.

The dimension of the trajectories constructed from the fibrillation signals depended on the length of the segments used. For the short, 1-s segments, the dimension approached approximately 5 ± 1 as embedding dimension was increased (up to 21). For the 8-s-long segments, the dimension approached approximately 8 ± 1. These results are consistent with those found by other researchers for human fibrillation.[14]

For segment lengths ≥ 4 s, no significant difference was found between the dimensions of the randomized signals and the fibrillation signals. In three of the four cases, this was true also for segments of length 1 and 2 s. In the fourth case, the randomized signal did have a significantly larger dimension than the fibrillation signal.

The inability to distinguish between fibrillation and the randomized signal on the basis of the dimensionality calculations suggests that the finite length of the signals is influencing the calculated dimension of fibrillation. The use of longer signals is problematic, since the condition of the heart is changing during fibrillation and stationarity of the signal cannot be assumed. An analyis of the repetition frequency of fibrillation suggests that fibrillation cannot be regarded as stationary for periods much longer than 10 s.[15]

CONCLUSIONS

The first step in testing whether a signal comes from a chaotic system is to construct a dynamical systems representation of the signal. That representation can then be tested directly for signs of chaos by calculating Lyapunov exponents,[16] or directly looking for the existence of a strange attractor or the stretching-and-folding dynamics that would lead to one.

We have been unable to construct a state-space representation of ventricular fibrillation that allows us to distinguish between VF and a similar, but random, signal. This means that the adequacy of the representation cannot be assured: the observed leveling off of dimension with increasing embedding dimension is due to the finite length of the signal, and not the existance of any attractor.

It may be that at a higher embedding dimension a significant difference would be found between fibrillation and a similar, but random, signal. It might turn out that in a high-dimensional embedding space, fibrillation is chaotic. The nonstationarity of fibrillation, however, precludes us from sensibly analyzing enough fibrillation data to test out this possibility. The possibility also exists that fibrillation is an example of a nonstationary transient, and that even if a dynamical system representation of fibrillation could be found, fibrillation might not be an attractor, but a transient.[15,17]

Although we cannot prove that fibrillation is not chaos, our results strongly suggest that, with the present data, it is not useful to regard fibrillation as chaos.

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REFERENCES

1. WIGGERS, C. J. 1940. The mechanism and nature of ventricular fibrillation. Am. Heart J. **20:** 399–412.
2. MOE, G. K., W. C. RHEINBOLDT & J. A. ABILDSKOV. 1964. A computer model of atrial fibrillation. Am. Heart J. **67:** 200–220.
3. SMITH, J. M. & R. J. COHEN. 1984. Simple computer model predicts wide range of ventricular dysrhythmias. Proc. Natl. Acad. Sci. U.S.A. **81:** 223–237.
4. KAPLAN, D. T., J. M. SMITH, B. E. H. SAXBERG & R. J. COHEN. 1988. Nonlinear dynamics in cardiac conduction. Math. Biosci. **90**(1): 19–48.
5. SMITH, J. M., E. A. CLANCY, C. R. VALERI, J. N. RUSKIN & R. J. COHEN. 1988. Electrical alternans and cardiac electrical stability. Circulation **77**(1): 110–121.

6. BERGE, P., Y. POMEAU & C. VIDAL. 1984. Order Within Chaos: Towards a Deterministic Approach to Turbulence. Wiley–Interscience. New York.
7. GUCKENHEIMER, J. & P. HOLMES. 1983. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer-Verlag. New York Berlin.
8. TAKENS, F. 1981. Detecting strange attractors in turbulence. *In* Lecture Notes in Mathematics, Vol. 898: 366. Springer-Verlag. New York/Berlin.
9. GRASSBERGER, P. & I. PROCACCIA. 1983. Characterization of strange attractors. Phys. Rev. Lett. **50**(5): 346–349.
10. LIEBOVITCH, L. S. & T. TOTH. A fast algorithm to determine fractal dimensions by box counting. In press.
11. KAPLAN, D. T. & R. J. COHEN. Is fibrillation chaos? In press.
12. BROOMHEAD, D. S. & G. P. KING. 1986. Extracting qualitative dynamics from experimental data. Physica D **20:** 217–236.
13. MAYER-KRESS, G. 1986. Dimensions and Entropies in Chaotic Systems: Quantification of Complex Behavior. Springer-Verlag. Berlin/New York.
14. RAVELLI, F. & R. ANTOLLINI. 1989. Analysis of the ventricular fibrillation ECG with methods from nonlinear dynamics. Computers in Cardiology.
15. KAPLAN, D. T. 1989. The dynamics of cardiac electrical instability. Ph.D. Thesis, Harvard University, Cambridge, Mass.
16. WOLFE, A., J. SWINNEY & J. VASTANO. 1985. Determining Lyapunov exponents from a time series. Physica D. **16:** 25–317.
17. CRUTCHFIELD, J. P. & K. KANEKO. 1988. Are attractors relevant to turbulence? Phys. Rev. Lett. **60**(26): 2715–2718.